

ROBUST MAPPING OF BRAIN SURFACE MESHES ONTO A UNIT SPHERE

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ABSTRACT

Complex shapes - such as the surface of the human brain - may be represented and analyzed in frequency space by a spherical harmonics transformation. A key step of the processing chain is introducing a suitable parametrization of the triangular mesh representing the brain surface. This problem corresponds to mapping the surface on a unit sphere. An algorithm is described that produces an optimal combination of an area- and angle-preserving mapping. A multi-resolution scheme provides the robustness required to map the highly detailed and convoluted brain surface.

Index Terms— magnetic resonance imaging, brain surface, morphometry, shape analysis

1. INTRODUCTION

It is a well-known fact in neuroanatomy that structural variants of the brain exist, some of which have to be considered as pathologic (e.g., macrogyria or microgyria) or abnormal (e.g., callosal agenesis) - but even normal brains exhibit a considerable variability. Brain shapes do not necessarily form a continuum in some descriptor space, but may cluster due to pre-determined genetical factors or acquired diseases.

An interesting approach for describing shapes is provided via a spherical harmonics transformation (SHT) that may be understood as a "Fourier transform on a sphere". Any surface of genus zero can be transformed into frequency space using a linear combination of basic shapes, and the resulting parameters of the shape spectrum can be used as a parsimonious shape description. The shape space spanned by spherical harmonic basis functions is orthogonal, and readily offers a metric for comparing and classifying shapes [2].

Implementing this apparently attractive approach poses difficult problems when highly convoluted objects with detailed structures are under study, such as the surface of the human brain. First, the brain's surface must be extracted from magnetic resonance (MR) imaging data of the human head, usually as a triangular mesh. This is considered a complex, but well developed methodology in medical image processing [8]. The "tricky" part is to ensure that the surface has a topological genus of zero [10] and is free of self-intersections [7].

For computing a SHT, a parameterization of the surface must be introduced which corresponds to mapping it onto a (unit) sphere. Such a mapping can be either angle- or area-preserving but not *isometric* (both) - which is most desirable because all properties of the surface are represented in the parameter domain. Finding an angle-preserving mapping yields an *almost unique* solution: all mappings form a so-called Möbius group [6]. Equiareal mappings are substantially different because from the point of view of uniqueness there are many more of them. In order to find a well-behaved mapping we need to aim for area-preservation with some minimization of angular distortion.

Current mapping approaches [1, 2, 6] that we tested fail for various reasons when applied to brain surface meshes of 100.000 triangles or more. This finding triggered the development of a mapping algorithm that is robust enough to handle large, convoluted brain surface meshes consisting of up to 500.000 triangles.

2. ALGORITHMS

Three steps are required to achieve a shape description by SHT: (1) extraction of the surface from imaging data as a triangle mesh, (2) parametrization of the mesh by mapping it onto a sphere, (3) computation of the SHT. We focus on the description of an algorithm to perform step (2), but outline steps (1) and (3) for completeness.

2.1. Surface Generation

Any method may be used to obtain a triangulated mesh representing the grey/white matter (GM/WM) interface. We briefly outline our procedure below and refer to [8] for a detailed description. T₁-weighted volumetric MR images were aligned with the stereotactical coordinate system and interpolated to an isotropical voxel size of 1mm using a fourth-order b-spline method. Data were corrected for intensity inhomogeneities by a fuzzy segmentation approach using 3 classes. A mask for the brain's WM was extracted from intensity class 2 by removing the outer hulls of the brain and cutting the brainstem at a level 15 mm below the posterior commissure. The resulting raw WM segmentation was edited to have a topological genus of zero (i.e., no holes or handles) [10]. A triangu-

lated surface was generated from the binary white matter segmentation using a variant of the "Marching Cubes" algorithm that preserves the original topology [3]. An improved representation of the GM/WM interface was obtained by treating the initial mesh as a deformable model [8] avoiding self-intersections [7]. Finally, we reduced the number of triangles to 100.000 by edge contraction [5] and split the surface mesh at the mid-sagittal plane to yield separate meshes representing the GM/WM interface of each hemisphere.

2.2. General notation

A parametric surface is defined by an one-to-one mapping $\phi : \Omega \mapsto \mathbf{R}^3$, with $\Omega \subset \mathbf{R}^2$ the parameter domain, and we denote ϕ a parameterization of the surface. In our case, this parameter domain are the longitude and latitude (u, v) of the unit sphere. More specifically, a parameterization of a surface S that has a triangulation is sought:

$$\mathcal{M} = \{[1 \dots n], \mathcal{T}, (s_i)_{i=1 \dots n}\}$$

where $[1 \dots n]$ denotes vertices at positions s_i and \mathcal{T} their set of corresponding triangles $\Delta_{\mathcal{M}}$. We require that the parameterization is piecewise linear so that ϕ maps vertices and triangles of \mathcal{M} onto vertices and triangles of the spherical triangulation \mathcal{M}' . The parameterization is found by minimizing a suitable error metric for the mapping over the parameter domain.

2.3. Error metric

The angle and area distortion error of a map $\phi|_{\Delta_{\mathcal{M}}}$ are given by:

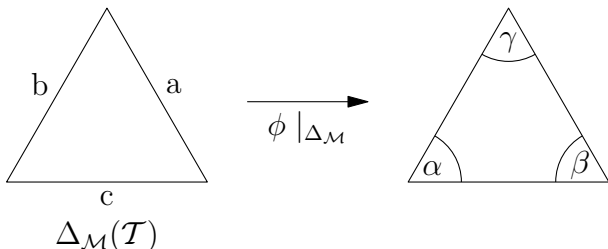
$$E_{angle} = \frac{\cot \alpha |a|^2 + \cot \beta |b|^2 + \cot \gamma |c|^2}{2 \text{area}(\Delta_{\mathcal{M}})},$$

$$E_{area} = \frac{\text{area}(\Delta_{\mathcal{M}'})}{\text{area}(\Delta_{\mathcal{M}})} + \frac{\text{area}(\Delta_{\mathcal{M}})}{\text{area}(\Delta_{\mathcal{M}'})} - 2.$$

Finally, the total distortion error of the map is:

$$E_{\phi} = \sum_{\Delta \in \mathcal{T}} E_{\Delta} = \sum_{\Delta \in \mathcal{T}} E_{angle} (E_{area})^{\theta} \text{area}(\Delta_{\mathcal{M}'}),$$

where θ varies between 0 and ∞ and controls the relative importance of angle and area preservation [4].



2.4. Optimization

The coordinates (u_i, v_i) of vertex i affect only those E_{Δ} for which i is incident with Δ . Thus, only the partial sum $E_i = \sum_{\Delta \in 1\text{-ring}(i)} E_{\Delta}$ is influenced by (u_i, v_i) .

Given an initial configuration $(u_i, v_i)_{i=1 \dots n}$, first all vertices are ordered by their error E_i . Then, each vertex i is optimized in (u_i, v_i) . The Simplex algorithm is used to solve this 2-parameter nonlinear optimization problem. The solution must be checked that (u_i, v_i) lies within the kernel of its 1-ring, so that triangles do not fold over. Minimizing E_i decreases E_{ϕ} in each step, so the algorithm is guaranteed to converge.

An initial configuration can be obtained by projecting vertices s_i to a position s'_i on the unit sphere, and by minimizing the distance of s'_i to all its adjacent vertices s'_j in the 1-ring:

$$s'_i = s'_i + \tau \sum_{j \in 1\text{-ring}(i)} |s'_j - s'_i|^2 \quad \text{with } \tau \in [0.10, 0.16]$$

This process is iterated until all overfolded configurations are resolved.

2.5. Introducing robustness

For simple surfaces (i.e. a cube) and for $\theta = 0$, the algorithm above converges for surfaces of up to 50k triangles. For complex surfaces (i.e., a brain) and other settings of θ , a multi-resolution scheme is employed. For surfaces with a small number of triangles, finding a solution (close) to the optimal one is more likely. The solution is propagated to the next more complex level, optimized again until a solution at the original resolution level is found.

The triangulated surface is subsampled by edge contraction. Each contraction step removes one vertex, two triangles at least three edges. A quadric error metric [5] guides the sequence of contraction steps with an additional constraint that avoids contractions leading to a topology change. Subsampling an orientable 2-manifold closed surface of genus zero ultimately leads to a tetrahedron.

The sequence of changes (vertex/edge/triangle removal) is recorded and "played back" for propagating a solution onto the next resolution level. Typically, a factor $f = [1.2 \dots 2.0]$ of faces are removed between levels. For our problem, starting at a lowest resolution level of 1000 – 5000 triangles is sufficient.

2.6. Computation of the SHT coefficients

A SHT of degree l_{max} has $p = (l_{max} + 1)^2$ functions. Given $(s_i, u_i, v_i)_{i=1 \dots n}$ points on the surface and their parameterization, we can assemble a $n \times p$ matrix B of complex-valued spherical harmonics $b_{i,j} = Y_l^m(u_i, v_i)$, where $j(l, m) = l^2 + l + m$, and a $n \times 3$ matrix $X = (s_i)_{i=1 \dots n}^T$ of the spatial coordinates. Typically, $p \ll n$, so the $p \times 3$ coefficient matrix C with row

entries $(c_l^m)^x, (c_l^m)^y, (c_l^m)^z$ is determined by least squares estimation:

$$C = (B^T B)^{-1} B^T X$$

To solve this large equation system on standard PCs, $B^T B$ and $B^T X$ are computed "on the fly". The surface approximation can be computed from these coefficients by the inverse transformation:

$$\hat{s}_i^{(t)}(u_i, v_i) = \sum_{l=0}^{l_{max}} \sum_{m=-l}^l (c_l^m)^{(t)} Y_l^m(u, v) \quad t = \{x, y, z\},$$

The distance $|\hat{s}_i - s_i|$ is a measure for the goodness-of-fit or reconstruction error.

3. EXPERIMENTS

We used a database of 513 MRI brain datasets of healthy volunteers. For demographic and acquisition details refer to [9]. Surfaces representing the GM/WM interface were computed from all datasets, yielding 1026 hemispheric surfaces. All surfaces were successfully processed by the algorithm described above with settings $\theta = 2, f = 1.3$. The processing time, including the SHT, was about 30 min on a standard workstation (AMD64, 2.4 GHz processor, Linux 2.6.15 operating system, 2 GB RAM). Neurobiological results will be reported in a separate publication, we merely focus on evaluating technical aspects of the algorithm here. By varying

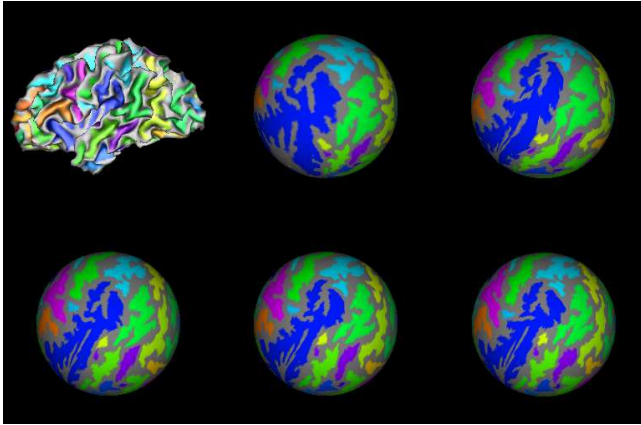


Fig. 1. Top left: View from left onto a white matter surface of a left brain hemisphere (50k triangles), where sulcal substructures are color-coded. Subsequent images show the corresponding mapping of this surface onto a sphere, for different settings of θ : 0.0, 0.5, 1.0, 1.5, and 2.0.

θ , the mapping shifts from angle to area-preservation. This is shown in Fig. 1: Sulci in an example hemisphere were color-coded and mapped onto a unit sphere with settings of θ between 0 and 2. The relative change in shape and size of the mapped sulci is impressive.

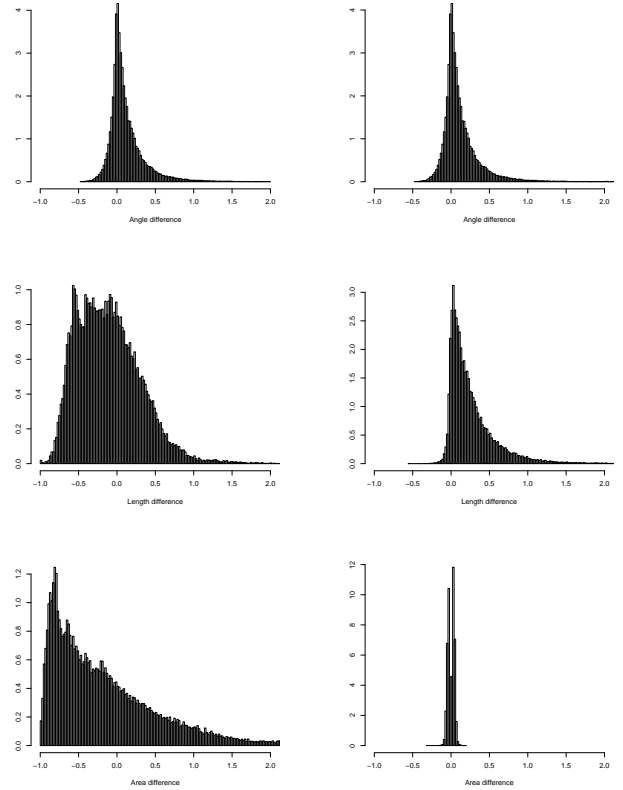


Fig. 2. Difference (as $v_{map}/v_{true} - 1$) for angles (α, β, γ , top), edge lengths (a, b, c , middle), and areas A (below). Columns correspond to different settings of θ : 0.0 (left), 2.0 (right).

In the next experiment, we studied the distribution of differences in angles, edge lengths and triangle areas induced by the mapping in dependence of θ . Due to limited space, results only for $\theta = \{0.0, 2.0\}$ are shown in Fig. 2. The effect of area preservation for $\theta = 2$ is readily apparent in the bottom figures. For $\theta > 1.0$, all distributions are sharply peaked, corresponding to a good preservation of angles *and* areas. Next, we were interested in the average reconstruction error of the GM/WM interface as a function of the maximum SHT degree l_{max} and varying settings of θ (circles: $\theta = 0.0$, triangles: $\theta = 1.0$, crosses: $\theta = 2.0$) (Fig. 3). To achieve a surface error that is in the order of the image resolution, $l_{max} \geq 30$ is required. Note that using an area-preserving mapping *always* leads to a better reconstruction resp. to a lower l_{max} . For our problem, choosing an area-preserving mapping requires 35% less coefficients to achieve the same reconstruction error. Finally, we show the reconstruction error (Fig. 4) color-coded on the GM/WM surface depending on l_{max} for $\theta = 1.0$, and $l_{max}=5$ (top left), 15 (top right), 25 (bottom left), 35 (bottom right). The color scale runs from blue (0 mm) to red (≥ 4 mm).

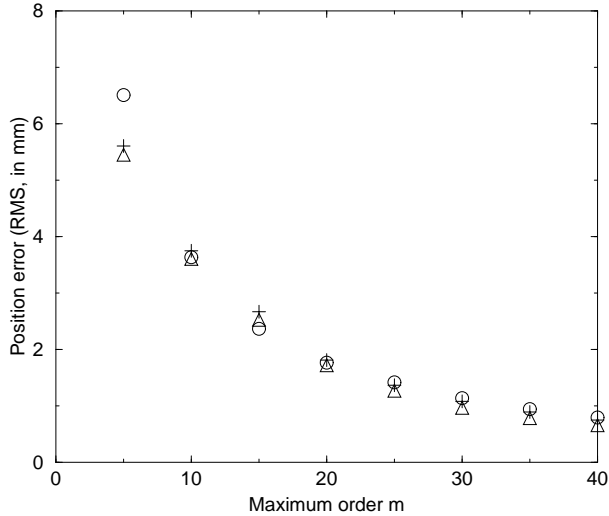


Fig. 3. Average reconstruction error of the GM/WM interface as a function of the maximum SHT degree l_{max} and varying settings of θ (circles: $\theta = 0.0$, triangles: $\theta = 1.0$, crosses: $\theta = 2.0$).

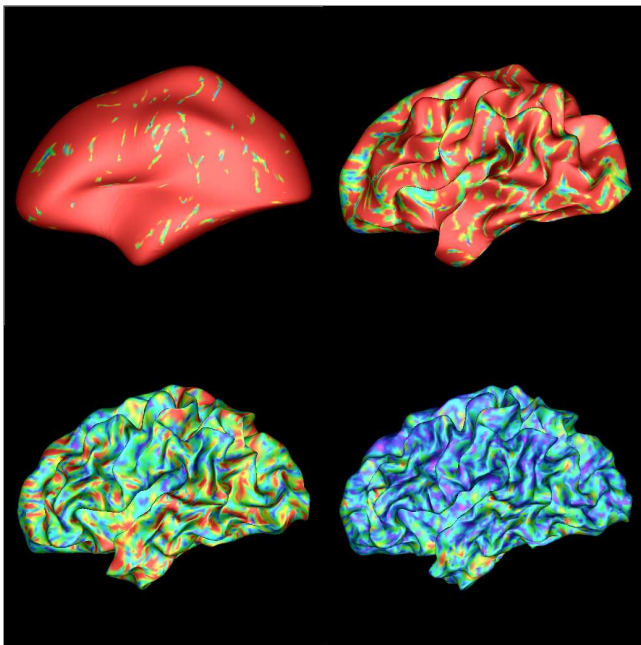


Fig. 4. Reconstruction error color-coded on the GM/WM surface depending on l_{max} .

4. DISCUSSION

The advantages of the proposed mapping algorithm are: (a) It is robust enough to handle large, highly convoluted meshes such as brain surfaces, with mesh sizes of up to 500.000 triangles. (b) By introducing an error metric that includes terms for area and angle-preservation, we achieve a mapping that

comes close to isometry. (c) Compared with an angle-preserving mapping, substantially less coefficients are required to represent the brain surface with same reconstruction error.

SHT coefficients are suitable for statistical analysis, such as a classification of brain shapes. The area-preserving mapping allows quantitative comparisons between brain regions across subjects. Shape models may be reconstructed to represent an average shape from a set. The sparse set of coefficients, and the coarse-to-fine representation are interesting options for building shape databases and the design of efficient similarity metrics and search strategies.

5. REFERENCES

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