Global Optimization Approaches to MEG Source Localization *

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Abstract

In this paper, we compare the performance of three typical and widely used optimization techniques for a specific MEG source localization problem. Firstly, we introduce a hybrid algorithm by combining genetic and local search strategies to overcome disadvantages of conventional genetic algorithms. Secondly, we apply the tabu search, a widely used optimization methods in combinational optimization and discrete mathematics, to source localization. To the best of our knowledge, this is the first attempt in the literature to apply tabu search to MEG/EEG source localization. Thirdly, in order to further comparison of the performance of above algorithms, simulated annealing is also applied to MEG source localization problem. The computer simulation results show that our local genetic algorithm is the most effective approach to dipole location.

Keywords: Magnetoencephalography (MEG), dipoles, global optimization, genetic algorithms, simulated annealing, tabu search.

1 Introduction

Measurements of the magnetoencephalography (MEG) as well as the electroencephalography (EEG) provide unique insights into the dynamic behavior of the human brain as they are able to follow changes in neural activity on a millisecond time-scale [5]. In comparison, the other functional imaging modalities (positron-tomography (PET) and functional magnetic resonance imaging (fMRI)) are limited in temporal resolution to time scales on the order of, at best, $100ms$ by physiological and signal-to-noise considerations. In the study of MEG/EEG, we are confronted with the following inverse problem. Given magnetic field values at a limited number of measurement points, we have to reconstruct the sources generating these data. Generally, given a suitable source and head model, this inverse problem can be cast as a nonlinear optimization problem of computing the location and moment parameters of the set of dipoles whose field best matches the MEG measurements in a least squares sense [11]. Mathematically, it is a very difficult nonlinear optimization problem because its objective function is very complex and always has many local optima, especially when the number of dipole source is large. Figure 1 shows such an example.

In order to solve this problem, various optimization techniques have been adopted. These optimization methods can loosely be classified into two groups: gradient based Newton-type methods such as Levenberg-Marquardt and gradient-free search methods such as the Nelder-Mead downhill simplex method. However, the gradient-based methods are problematic for this specific problem because they will easily be trapped by the local optima, which probably results in incorrect estimates of the dipole parameters. Though Nelder-Mead downhill simplex method is better than the Newton-type technique in escaping local optima, Khosla *et al.* [7] demonstrated that it is sensitive to starting parameter estimates and can also converge to a suboptimal local optimum. Therefore, conventional gradient based optimization methods and Nelder-Mead downhill simplex method are hardly suitable to MEG source localization. The key requirement to any global optimization method is that it must be able to escape in local minima and continue the search to give a near-optimal final so-

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lution whatever the initial conditions. *Simulated annealing* (SA) and *genetic algorithm* (GA) meet this requirement, and majority of work along this stream focused on how to apply simulated annealing algorithm as an alternative to conventional gradient-based optimization methods. Unfortunately, K. Uutela *et al.* [12] have shown that for the dipole location, the performance of SA is the worst one among their chosen three types of global optimization. Besides GA and SA, several other global optimization techniques exist, the most popular in them is *tabu search* (TS) [3]. There are very few works for applying it to continuous problems [6]. It has also been shown that stimulated annealing is a special case of tabu search [3]. A key feature of this algorithm is a tabu list included in the search process. This provides TS with some memory and endorses some intelligence to find the optimal solution. So it is reasonable to expect that TS should be superior to SA for the dipole localization. It remains natural to ask whether GA is the most effective algorithm. Moreover, in [12] only conventional GA was adopted, though some new features have been added to GA in its implementation. However, it has been shown that the conventional GA has a very poor local performance because of the random search of GA.

Although a lot of different optimization methods exist, the efficacy of an optimization method is always problem specific. In this paper, we compare the performance of three typical and widely used optimization techniques for a specific MEG source localization problem. The computer simulation results show that our local genetic search algorithm is the most effective approach to dipole localization.

The paper is organized as follows. Section 2 devotes to the formulation of the problem. In section 3, we give a detailed description of genetic algorithm, stimulated annealing, and tabu search and their implementations. In Section 4, we describe the procedure of computer stimulation. Section 5 is experimental results. The final section is conclusion.

2 MEG Source Imaging as an Inverse Problem

Given a suitable source and head model, we want to reconstruct the sources from some given magnetic field values at limited number of measurement points. This inverse problem can be cast as the following least squares problem of estimating the location and moment parameter of current dipoles [11].

$$E(L,Q) = \|B - G(L)Q\|_F^2 \tag{1}$$

where B are current dipoles, L are location parameters, Q are dipole moments, $G(L)$ is the gain matrix, and $\|\cdot\|_F$ indicates the Frobenius norm. We refer to [11] for a detailed derivation of this formula. Thus the inverse problem is to find the set $\{L, Q\}$ to minimize this error function. It is obvious that this is a nonlinear optimization problem. So we can only use some iterative optimization algorithm to solve it. Assuming that there exist N dipoles and k time points, then there will be $3N$ location parameter and $3Nk$ moment parameters, for an overall of $3N(k+1)$ parameters. In practice, it is a very difficult optimization problem only due to its dimension. For example, if there are $N = 3$ dipoles and $k = 80$ time points, then we will have to search a 729 dimensional parameter space to find the global optima for this problem.

Fortunately, the computational complexity of the problem can be greatly reduced by separating the linear and nonlinear parameters because B is linear function of parameter Q. The method to factor out the linear moments has been used by many researchers, say, [11] and references therein, and has been mathematically justified in Golub *et al.* [2]. First, we assume that we know the locations L, then a solution for matrix Q that will minimize $E(L,Q)$ is

$$Q = G^* B \tag{2}$$

where G^* is Moore-Penrose pseudo-inverse which can be found by $G^* = V\Sigma^+U^T$, where $G = U\Sigma V^T$ is the singular value decomposition(SVD), and Σ^+ is the inverse of Σ. Replacing Q with this pseudo-inverse solution before solving for L, the cost function of the inverse problem then becomes

$$E = \|B - GQ\|_F^2 = |B - GG^*B\|_F^2 = (I - GG^*)B\|_F^2 \tag{3}$$

Now we can see the cost function depends only on the gain matrix G, which is a function of only $3N$ nonlinear location parameters. The number of parameters is reduced to 9 in the case of three dipoles.

It has been shown by Mosher *et al.* [11] that the computation process can be further simplified by using some technique to compute $I - GG^*$, and the final results is as follows.

$$E = \|P_G^\perp B\|_F^2 = |U_{m-r}^T B\|_F^2 \tag{4}$$

where $P_G^\perp$ is projections which project the data into left null space of G. And supposing $G = [U_r U_{m-r}]\Sigma V$. For details, we refer to Mosher *et al.* [11].

After finding the value of L using some nonlinear optimization technique, $3Nk$ moment parameters in Q can be estimated using equation (2). This is computationally cheap, because only the pseudo-inverse of G needs to be computed.

3 Optimization Methods

In section 2, the problem of MEG source localization is addressed as an optimization problem. Once it is understood that MEG source localization is such an optimization problem, the use of any technique for tackling optimization problem suggests itself. In this section, we will introduce the genetic algorithm, stimulated annealing, and tabu search and then give details of our implementation to handle MEG source localization problem.

3.1 Genetic Algorithm

Genetic algorithms (GA) are capable of searching for global optimum in functions which cause difficulty for gradient based methods [4]. They have been successfully applied to many nonlinear, multi-peak, continuous or combinatorial optimization problems. Principal advantages of GA are domain independence, non-linearity and robustness. Domain independence means that it is easy to write one general computer program for solving many different optimization problems. GA does not require linearity, convexity, differentiability which are necessary for many conventional optimization techniques. The only requirement for GA is to calculate some objective function, which may be highly complicated and non-linear. The above two characteristics of GA assure that this method is inherently robust. For details about GA, we refer to [4].

K. Uutela *et al.* [12] have applied GA to the problem of MEG source location. They adopted the conventional GA though some properties of SA and c-means have been used for GA in their implementation. However, conventional GA has a very poor local performance because of the random search of GA. To get a good solution, great computational costs are inevitable. Improvements can be made in methodological decisions and parametric choices to enhance the performance of GA. In this paper, several remarkable features are added and some important extensions are also made to improve the performance of the conventional GA.

(1) Hybrid Algorithm In many cases, hybridizing GA with another heuristic can significantly enhance the effectiveness of a GA [9]. Since GA has poor local search performance and conventional local search methods have remarkable ability in finding local optima, we propose a hybrid algorithm, which combines GA with a modified local search procedure. The local search is also applied to elite solutions inherited from the previous populations. Some parameters are introduced into the algorithm to control the local search. The parameter *Step* is used to control the initial step of the search. The parameter ε is used to control the distance from the solution to its nearest local optimum. If we use a large value of *Step* and ε, the local search will be fast and the computational cost will be little though the solution will not be precise. This is designed at the beginning of the algorithm. At the end of the optimization, a high precise solution is needed, we should set *Step* and ε to be small.

(2) Elitist Strategy Two sets of solutions are stored in our algorithm: a current population and an elite set. After genetic operation and local search, the current population is replaced with the improved population and the elite set is updated from the new population. A local search procedure is applied to solutions in the elite set. By preserving good solutions, we can avoid losing some excellent solutions. It has been shown that only after one adopts the elitist strategy, the GA can mathematically converge to a global optimum [4]. In our algorithm, the parameter *EliteRate* is used to control the number of the elite. A careful choice of *EliteRate* is necessary in order to provide good convergence properties. Otherwise, one or two outstanding solutions will rapidly be dominant in the group and will lead to premature convergence of the algorithm.

(3) Selection Operation and Crossover Operation In our algorithm, we adopt the roulette wheel selection mechanism. The selection probability $P(x)$ in the current population Φ can be expressed as: $P(x) = f^{\lambda}(x)/\sum_{x\in\Phi} f^{\lambda}(x)$, where $f(x)$ is the fitness value of solution x and λ is the parameter to control the scaling procedure of selection operation. Different values of λ can make GA result in different solutions: when the value of λ is large, the selection mechanism is strong and competition in the group is intense, some outstanding solutions in the population have greater chance to survive. However, the GA is easily trapped in a local optimum because one or two outstanding solutions in the group will be dominant in the population rapidly and the evolution will stop at a sub-optimal solution. Thus in the beginning of GA, we give λ a small value to limit the competition, and we increase λ in subsequent generations in order to stimulate the evolution. Another reason for us to adopt a scaling strategy is that the difference of the fitness value of different solutions may be very large for MEG source localization problem.

(4) Parameters *EliteRate*, λ, ε, *Step*, and *GenerationNumber* in our algorithm The performance of the algorithm largely depends on the parametric choice. In order to reduce the computational cost, simultaneously to keep the convergence to a global optimal solution, we have to carefully adjust the parameters in our algorithm in different stages of the algorithm. We divide the algorithm into three stages and adjust the parameters in each stage. To ensure convergence to a global optimum, the competition is limited early on, but it is stimulated later. The number of elite is limited at the start and increased later. To reduce computational costs, parameters of the local search were optimized since the main computation is spend on local search.

3.2 Simulated Annealing

Simulated annealing (SA) is a stochastic simulation method originally proposed by Kirkpatrick *et al.* [8]. It is a simple and robust algorithm and proved to be useful in wide range of complex combinatorial optimization problems. The idea of SA mainly comes from the field of statistical mechanics. The process of annealing is analogous to the optimization process, in which the value of the cost function take the role of the energy of the system, and the global optimum corresponds to the energy of the ground state of the system. Metropolis *et al.* [10] proposed a Monte Carlo method to simulate the evolution of the system for a fixed value of temperature T. If the system in energy state E_1 is perturbed to another energy state E_2, the new state is accepted with the probability $\exp(-\Delta E/kT)$, where $\Delta E = E_2 - E_1$, that is to say when the perturbed state is of lower energy, it will always be accepted. When perturbed to a higher energy state, the probability to accept the new energy state will depend on ΔE and T. By repeating the basic step many times, the system will evolve into thermal equilibrium and the temperature is lowered. As temperature decreases, the probability of accepting uphill steps will decrease and the algorithm will eventually converge into a global optimum. We refer to [8] for the details about SA.

Because of its outstanding performance in finding global optimal solution, SA has been implemented in different ways to MEG/EEG source localization problem. Therefore, we will make a comprison of the performance of SA with others.

3.3 Tabu Search

Tabu search is effective for many optimization problems [3]. It is a meta-heuristic method that guides a local heuristic search procedure to explore the solution space beyond local optimality. It is different from the well-known hill-climbing local search techniques because the tabu search allows moves out of a current solution that makes the objective function worse in the hope that it eventually will achieve a better solution. It is also different from the simulated annealing and genetic algorithms because the tabu search includes a memory mechanism.

Tabu search starts with a certain solution x_{now}, followed by a certain set of candidate moves. A candidate set of moves $\mathrm{Cand}(x_{now})$ can be obtained from its neighborhood $N(x_{now})$. Based on the history record of TS, some of the moves are tabu, some of the moves are permitted. Then the aspiration criteria is applied to the candidate set, and each solution in the set is evaluated based on its value of cost function $c(x)$, the history record H, and the aspiration criteria. The best move in the set is selected to be the new solution x_{next}.The procedure is repeated until some stop criteria is satisfied.

To the best of our knowledge, this is the first attempt that tabu search is used to solve the MEG source localization problem. Conventional applications of TS mostly focus on combinatorial optimization problems. However, the MEG/EEG source localization problem is a continuous optimization problem. In order to apply TS effectively to it, we have to carefully devise some strategies specific to this problem. These strategies include the selection of suitable move attributes, specific intensification and diversification strategies using recency-based memory and frequency-based memory, and some suitable aspiration criteria.

4 Computer Simulations

In order to find the relative merit of the proposed optimization methods, we carefully devise some experiments and give some mathematical measure to compare performance of these algorithms. The performance of proposed algorithms was assessed and compared via a large number of computer simulations. In this section, we describe the procedure of our simulation in detail.

4.1. Assumptions A current dipole model and a spherically symmetric conductor head model are adopted in our simulation. The human head is assumed to be a spherically symmetric conductor with an outer radius of 120 mm. The magnetic field data are measured by 17 evenly distributed sensors. Only the magnetic field component normal to the spherical surface is measured, thus only magnetic fields due to the primary tangential dipole currents were computed, as we have discussed in Section 2. Our simulation can be easily extended to some more complex forward model. We should note is that the number and location of the detectors can significantly influence the difficulty of source localization.

In the simulation, three dipoles ($N = 3$) are assumed to generate magnetic data. Since the radial component of a current dipole does not generate a measurable magnetic field on a spherical surface, this component is neglected and only the location and two tangential components are considered. Two sets of data which have different localization, orientation, and time course were used. Two cases are designed to evaluate the performance of these algorithms when the cost function is different. In Case 1, dipoles are separated by a large distance, while in Case 2, they are close. The problem of escaping local optima is expected to be severe in Case 2. We assume that the three dipoles are all with fixed location and orientation. The dipole parameters are listed in TABLE I. A total of 20 time samples are generated in our simulation. The shape of the dipole time courses is assumed to be double-sided Gaussian function since physiologically it is expected the wave forms will have smooth shapes like the ones chosen here.

TABLE I

Dipole parameters to generate magnetic field in **Case 1**

	L_x	L_y	L_z	M_θ	M_ϕ
Dipole 1	2.8	-1.7	8.3	0.5	-0.5
Dipole 2	-2.9	8.3	0.0	0.2	0.5
Dipole 3	8.1	3.3	-1.2	0.7	0.3

Dipole Parameters to generate magnetic field in **Case 2**

	L_x	L_y	L_z	M_θ	M_ϕ
Dipole 1	2.8	-1.7	8.3	0.5	0.5
Dipole 2	-2.9	-1.6	8.3	0.5	-0.5
Dipole 3	0	3.3	8.4	-0.5	-0.5

It should be noted that the emphasis of our work in this paper is to compare the performance of different optimization techniques when applied to the MEG/EEG source localization problem. Thus we do not consider the effect of noise in the simulated magnetic field data because the global minimum may be affected by noise. Thus, we cannot tell if the failure to find global optimum is the effect of noise or the optimization method. In this paper, the number of parameters is assumed to be known. We do not consider the unknown number case so that we can focus on the comparison of the relative performance of different optimization techniques.

4.2. Methodology for Evaluation of Optimization Algorithms Evaluating different optimization algorithms is difficult task. The relative performance of different algorithms is task specific. In a specific implementation of an algorithm, some choices have to be made on its many parameters. Such choices affect the performance of the algorithm. Thus, in our experiments, we first use an objective evaluation and optimization procedure to give the best parameter choices for a particular task, then we compare the efficacy of different algorithms.

To compare different algorithms, some mathematical measures are needed. In this paper, we use localization error d as a measure for each the precision of dipole estimate, which is defined as follows.

$$d = \sqrt{(\bar{L}_x - L_x)^2 + (\bar{L}_y - L_y)^2 + (\bar{L}_z - L_z)^2}. \quad (5)$$

We use the following criterion to determine whether estimated locations of the three dipoles are correct. When we obtain a solution, we calculate the localization error d_i for each dipole. We accept a solution when the localization error of all of the three estimated dipoles is less than 0.05 cm, i.e., $d_i \le 0.05$, $i = 1, 2, 3$.

4.3. Evaluate the Dipole Moment and Dipole Time Courses Having estimated the dipole location L, the gain matrix G can be calculated using the forward model. Q can be obtained using $Q = G^* B$. Then the moment and time courses of each dipole can be easily obtained by finding the best rank one approximation for each dipole. This has been described in detail in Mosher *et al.* [11]. In this paper, we give one example to calculate the dipole time courses using a correct estimation of L obtained in GA.

4.4. Simulation Procedures Our simulation consists of the following steps:

(1) Compute the magnetic field using the forward model and dipole parameters as stated above.

(2) Estimate source locations using GA, SA and TS algorithms respectively by minimizing the cost function as we described in Section 2.

   1) Evaluate the performance of each algorithm when the parameter configuration is different and get the optimal parametric choice for each algorithm.

   2) Compare the performance of different algorithms when the computational cost of each algorithm is same.

(3) Using the estimated source location, compute best-fit dipole time course for each dipole.

5 Experimental Results

5.1. Genetic Algorithm There are six parameters in GA and we selected the optimal parameter configuration from a large number of simulations. In our experiments, the parameters *EliteRate*, *Step* ε, and λ have been selected according to the configuration in TABLE I. While we give different values to *Npop* and *GenerationNumber*, and do 100 experiments for each configuration and test the probability it can find the correct location and the localization error d. The results are listed in TABLE III in the last page.

TABLE II
Configuration of parameters in different stages of the hybrid algorithm. LS means local search.

	EliteRate	*Step*	ε	λ
Satge 1	0.1	No LS	No LS	0.3
Satge 2	0.15	2	0.5	0.6
Satge 3	0.3	0.5	0.05	1

Our results show that GA is capable to find the correct dipole locations in all cases when the population size and generation number is large enough. Our implementation of GA is very efficient and effective to localize the dipole, since the population in our algorithm is rather small compared with the previous implementation of GA to this problem [12]. Our results illustrate that the strategy of hybridizing conventional GA with local search is very suitable to this kind of MEG source localization problem. In case 1, the performance of GA is so remarkable that it can find global optimum with a large probability even the computational cost is very small. We think that this results from local search feature of our algorithm. In Case 2, the estimation of dipole 3 is correct, while the estimated location of dipole 1 and dipole 2 always has an error of 2 cm. Only when the computational cost is rather large, the probability to find the correct location is large. We think this is probably because the three dipoles in case 2 are closely related. Drawing the cost function in the vicinity of correct location we find two local minima in which solutions are trapped. The problem is very ill-posed in case 2. The global optimum of cost function is a very small and very deep hole in a flat area with some large size local optima beside it. Thus the algorithm has greater possibility to go into the local optima since the area of the peak of local optima is larger than the real optimum. GA has remarkable global performance in this problem since it can find the global optimum of this function if the computation cost is large.

5.2. Simulated Annealing The most important parameters in SA are T_0, α and L_k. A suitable configuration of the parameters in SA is a key factor for its successful implementation. We performed a large number of experiments using different parameter sets to test the performance of SA. In our simulation, T_0 is set at 0.4 and the start location is randomly specified in the parameter space. For each parameter pair, 100 simulations were conducted and the results are given in TABLE IV in the last page.

We can see from TABLE IV that SA can also find the global minimum in most cases if we use a suitable annealing schedule and a sufficiently large Markov chain. If the length of Markov chain is not long enough the possibility to find the global optimum is little. Results also show that SA requires great computational costs. Again, the probability to find the correct solution in Case 2 is less than in Case 1, given the same computational cost.

5.3. Tabu Search The key factor for performance of TS is the number of iterations. The more the number of iterations, because more points in parameter space are examined. TS have search. We performed some experiments to test the influence of iteration number on the results of TS. The results are listed in TABLE V in the last page.

The performance of TS is rather good for the MEG source localization problem. Given the suitable search strategy and high number of iterations, the possibility to find the correct dipole location is almost 1. Even in Case 2, TS has a rather high success rate and the computation cost is small. If we devise suitable strategies aimed at the specific function, TS exhibits excellent performance. The greatest difficulty to apply TS to a problem is probably find a proper search strategy and efficiently using memory.

5.4. Comparison of Three Methods GA, SA and TS, all have the potential to find the global optimum if their parameters are correctly configured and enough computational cost is spent. GA demands that the population and the number of generations is large enough. SA requires the decrease of temperature is slow enough and the length of Markov chain is large enough. TS requires the iteration number is large enough. However, in practice, the resource of computation is limited. In order to compare the performance of them, we must limit the computational cost. We restrict the evaluated points to 10000, 20000, 100000, respectively and use each algorithm to estimate the dipole location. The algorithm, which has greater probability to give the correct estimation, is deemed to be superior to others. The results are listed in TABLE VI.

TABLE VI
Possibility to find the correct dipole location using genetic algorithm, simulated annealing and tabu search when the computational cost is limited. The parameters of each algorithm have been optimized by a specific procedure.

Iteration Number		GA	SA	TS
Case 1	5,000	73%	34%	68%
	10,000	95%	64%	96%
	50,000	100%	98%	97%
	100,000	100%	100%	100%
Case 2	20,000	26%	23%	24%
	50,000	53%	36%	28%
	100,000	68%	55%	39%
	200,000	94%	75%	49%

In Case 1, when the dipoles are spatially and temporally distinct, the performance of all algorithms is good. GA and TS perform slightly better than SA. The hybrid algorithm has excellent local performance which will make the solution close up to the global optimum rapidly. TS is also suitable to this problem, and a suitable search strategy will make the algorithm very efficient. In Case 2, the three dipoles are very close in space and time. All these algorithms have some difficulty to find the correct estimation, and a greater number of estimation is needed to find the global optimum. GA performs best followed by SA proving that GA and SA have remarkable ability to find the global optimum. TS, however, is trapped by local optima. TS can be improved by adopting strategies specific to this problem. Similarly, GA is the most effective optimization algorithm. It can be considered as an alternative to the prevalent methods for solving the MEG source localization problem. Simulated annealing has also the potential to give the correct estimation even when the problem is very difficult. But simulated annealing has larger computation of demands than the genetic local search algorithm. Hybridizing SA with another heuristic may improve the efficiency of SA. Though tabu search needs less computation and performs nicely in simple problems, it is not as good as GA and SA for this special problem.

6 Conclusions

In this paper, We have first introduced a hybrid algorithm by combining genetic algorithm with local search techniques in order to overcome disadvantages of the conventional genetic algorithms. We have also applied tabu search, a widely used optimization methods in combinational optimization and discrete mathematics, to MEG source localization. To the best of our knowledge, this is the first attempt in the literature to apply tabu search to MEG/EEG source localization. Finally, we compared the performance of our genetic algorithm, tabu search, and stimulated annealing, for a special source localization. The computer simulation results show that our local genetic algorithm is the most effective approach to dipole location. We noted that the performance of tabu search for MEG source localization is not as good as what we expected. That is, it is generally believed that tabu search should have a better performance for continuous problem compared with stimulated annealing [1]. We have not figured out whether our parameter choices for tabu search result in this phenomenon. The further work along this stream is ongoing. Our method in this paper is also suitable for EEG source localization.

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TABLE III

Results using genetic local search algorithm on noiseless data in Case 1 and Case 2. The parameters of our algorithm is configured as stated in Section 3. Initial population is randomly generated in the parameter space. The criteria for success and the measurement d_i are defined in Section 4.

		Test 1	Test 2	Test 3	Test 4
Npop		5	15	30	50
GenerNum		10	25	45	80
Case 1	Success	65%	95%	100%	100%
	d_1 cm	0.018 ± 0.03	0.008 ± 0.02	0.009 ± 0.01	0.014 ± 0.01
	d_2 cm	0.35 ± 0.016	0.057 ± 0.09	0.035 ± 0.07	0.042 ± 0.04
	d_3 cm	1.07 ± 1.42	0.03 ± 0.003	0.047 ± 0.032	0.021 ± 0.01
Case 2	Success	23%	35%	64%	93%
	d_1 cm	2.49 ± 1.26	1.40 ± 1.37	1.11 ± 1.34	0.24 ± 0.37
	d_2 cm	2.11 ± 1.96	0.92 ± 1.34	0.61 ± 1.17	0.17 ± 0.23
	d_3 cm	0.18 ± 0.63	0.03 ± 0.01	0.01 ± 0.005	0.01 ± 0.01

TABLE IV

Results using simulated annealing on noiseless data in Case 1 and Case 2. The parameters of opur algorithm is configured as stated above. Initial location is randomly generated in the parameter space.

		Test 1	Test 2	Test 3	Test 4
α		0.9	0.9	0.9	0.9
L_k		200	500	1000	2000
Case 1	Success	89%	97%	100%	100%
	d_1 cm	0.02 ± 0.01	0.02 ± 0.01	0.01 ± 0.01	0.01 ± 0.01
	d_2 cm	0.05 ± 0.02	0.03 ± 0.02	0.02 ± 0.01	0.01 ± 0.01
	d_3 cm	0.04 ± 0.015	0.03 ± 0.01	0.02 ± 0.01	0.01 ± 0.01
Case 2	Success	29%	41%	69%	83%
	d_1 cm	2.31 ± 6.44	1.59 ± 1.13	1.87 ± 1.23	0.43 ± 0.84
	d_2 cm	2.25 ± 1.57	1.74 ± 1.02	1.04 ± 1.37	0.36 ± 0.63
	d_3 cm	0.35 ± 0.23	0.21 ± 0.20	0.09 ± 0.07	0.04 ± 0.01

TABLE V

Results using tabu search on noiseless data in Case 1 and Case 2. The strategies of our implementation of the algorithm is as stated in Section 3. Initial location is set as the original point in the parameter space. The criteria for success and the measurement are defined in Section 4.

		Test 1	Test 2	Test 3	Test 4
Iteration Nummber		200	500	700	1000
Case 1	Success	40%	62%	80%	94%
	d_1 cm	0.09 ± 0.07	0.02 ± 0.04	0.02 ± 0.02	0.02 ± 0.01
	d_2 cm	0.33 ± 0.27	0.29 ± 0.23	0.14 ± 0.17	0.11 ± 0.14
	d_3 cm	0.42 ± 0.51	0.39 ± 0.38	0.27 ± 0.33	0.18 ± 0.10
Case 2	Success	23%	33%	35%	39%
	d_1 cm	0.57 ± 0.49	0.95 ± 1.27	0.12 ± 0.03	0.02 ± 0.01
	d_2 cm	1.36 ± 1.46	1.03 ± 1.36	1.06 ± 1.57	1.15 ± 1.97
	d_3 cm	0.08 ± 0.06	0.05 ± 0.02	0.05 ± 0.02	0.06 ± 0.01